45 $\lim_{x \to 0^+} (1+x)^{\frac{1}{x}}$

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8.3 Relative Rates of Growth

Definitions: Faster, Slower, Same-rate Growth

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \infty$ f is faster than g

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$ g is faster than f

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = L$ f, g same

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$y = x$, $y = 2x$ $\lim_{x \to \infty} \frac{x}{2x} = \frac{1}{2}$

$\boxed{\text{double } x \text{ double } y}$ $y = 2x$, $y = 2 \cdot 2x$

$y = x$, $y = x^2$ $\lim_{x \to \infty} \frac{x}{x^2} = \lim_{x \to \infty} \frac{1}{x} = 0$

$y = (2x)^2$

$y = 4x^2$

double x

quadruple y

x^2 faster than x

x slower than x^2

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Which function grows faster?

e^x, x^2

$\lim_{x \to \infty} \frac{e^x}{x^2} = \lim_{x \to \infty} \frac{e^x}{2x} = \lim_{x \to \infty} \frac{e^x}{2} = \infty$

$\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$

So e^x faster than x^2

$\ln x, x, x^2$

$\lim_{x \to \infty} \frac{\ln x}{x} = \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = 0$

$\ln x$ slower

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Show the functions grow at the same rate

$x, x + \sin x$

$\lim_{x \to \infty} \frac{x + \sin x}{x} = \lim_{x \to \infty} \left(1 + \overset{\to 0}{\frac{\sin x}{x}}\right) \le \left(1 + \frac{1}{x}\right) = 1$

$\log_a x, \log_b x$

$\lim_{x \to \infty} \frac{\log_a x}{\log_b x} = \lim_{x \to \infty} \frac{\frac{\cancel{\ln x}}{\ln a}}{\frac{\cancel{\ln x}}{\ln b}} = \frac{\ln b}{\ln a}$

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Transitivity of Growing Rates

Show the functions grow at the same rate by comparing both with x

$\sqrt{x^2+5}, (2\sqrt{x}-1)^2$

$\downarrow$ $\downarrow$

$\sqrt{x^2}$ $(\sqrt{x})^2$

$\downarrow$ $\downarrow$

x x

$\lim_{x \to \infty} \frac{\sqrt{x^2+5}}{x}$ same

$\lim_{x \to \infty} \frac{\sqrt{x^2+5}}{\sqrt{x^2}}$

$\lim_{x \to \infty} \sqrt{\frac{x^2+5}{x^2}}$

$\lim_{x \to \infty} \sqrt{1 + \frac{5}{x^2}} = 1$

$\lim_{x \to \infty} \frac{(2\sqrt{x}-1)^2}{(\sqrt{x})^2}$

$\lim_{x \to \infty} \left(\frac{2\sqrt{x}-1}{\sqrt{x}}\right)^2$

$\lim_{x \to \infty} \left(2 - \frac{1}{\sqrt{x}}\right)^2 = 4$

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Sequential vs Binary Searches

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